	MIDTERM EXAM 1	
Name, Surname:	Department:	GRADE
Student No:	Course: Linear Algebra	
Signature:	Exam Date: 03/04/2019	

Each problem is worth equal points. Duration is 75 minutes.

1. For what values of r and s is the linear system $\begin{aligned} x+y+z&=1 \\ x \quad +3z&=-2+s \\ x-y+rz&=3 \end{aligned}$ inconsistent?

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & -2+s \\ 1 & -1 & r & 3 \end{array}\right] \quad \begin{array}{c} -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -3+s \\ 0 & -2 & r-1 & 2 \end{array}\right]$$

$$-2r_2+r_3 \to r_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -3+s \\ 0 & 0 & r-5 & 8-2s \end{array}\right]$$

$\boxed{r=5\ ,\ s\neq 4}$

2. (A) For what values of a, is the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & a & 1 \end{bmatrix}$ invertible?

$$\det(A)=\begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -4 \\ 0 & a-4 & -5 \end{vmatrix} = 1\left(15+4(a-4)\right) = 4a-1$$

$\boxed{a\neq 1/4}$

(B) Suppose that $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & a & 1 \end{bmatrix}$ is invertible. Find the (2,1) entry of the inverse of A.

$$\left(A^{-1}\right)_{2,1} = \frac{1}{\det A} A_{12} \qquad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = -(2-4) = 2$$

$\boxed{\dfrac{2}{4a-1}}$

3. Let $ad - bc = 2$ and $c = 2a - 4$. Find the value of y determined by the linear system $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad y = \frac{1}{ad-bc}(-c+2a) = \frac{4}{2}$$

$\boxed{2}$

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4. The linear system $C^T A^{-1}\mathbf{x} = \mathbf{b}$ is such that A and C are nonsingular, with $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $C^{-1} = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the solution $\mathbf{x}$.

$$x = A(C^T)^{-1} b = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$\boxed{\begin{bmatrix} 8 \\ 6 \end{bmatrix}}$

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5. If $\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = 4$, then find $\det\begin{bmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{bmatrix}$

$$\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & \cancel{2}c_3 - c_2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ c_1 & c_2 & 4c_3 - 2c_2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & a_2 & 4a_3 \\ b_1 & b_2 & 4b_3 \\ c_1 & c_2 & 4c_3 \end{vmatrix}$$

$$= \frac{1}{2}\cdot 4\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \frac{1}{2}\cdot 4 \cdot 4$$

$\boxed{8}$